Point Source Turbulent Diffusion in a Pipe

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Turbulent diffusion from a small source located in the center of a 3-in. pipe was studied. Hydrogen and carbon dioxide were mixed in air, and potassium chloride solution was mixed in water. Eddy diffusion coefficients calculated from the data are correlated by plotting $(E)/(2aU_c)$ vs. N_{Re}. Differences from other data in the literature are discussed. The results could not be used for determining the form of the correlation coefficient because of inaccuracies in the measurements for small diffusion times.

Many processes involving turbulent transport of heat and mass may be described in terms of a number of sources and sinks of heat or mass in the flow field being considered. For example in the transfer of heat from a hot wall to a cold wall through a turbulently flowing fluid the hot wall may be considered as consisting of a series of infinitesimal heat sources, and the cold wall may be considered as consisting of a series of infinitesimal sinks of heat (*7*). A better understanding of turbulent transfer processes therefore may be obtained by studying the behavior of sources of heat and mass in a turbulent field. In this paper data are presented on the diffusion of mass from a small source located in the center of a pipe through which water or air is flowing.

A foreign substance is injected into the flow through a small tube. Samples are withdrawn from the flow stream at different positions along a diameter in a plane downstream of the source plane over a time period which is long compared with the time scale of the turbulence and are analyzed for the concentration of the diffusing substance. The spread of the diffusing material is characterized by the x-component of the mean-squared displacement of the material from its centroid, designated by the symbol $\overline{X^2}$. By carrying out such measurements for different distances between the plane of the injector and the plane of the sampling tube the rate of spread of the diffusing material can be obtained. The data presented in this paper are compared with results of investigations by Towle and Sherwood (*18*) and by Mickelsen (*14*) for air flows in a pipe. Disagreement amongst the three investigations is noted, and possible explanations are discussed.

THEORY

Taylor's statistical description of turbulent diffusion (*17*) is used to present the results of this investigation. This treatment has been described in an excellent review by Frenkiel (*6*) and in numerous other articles which have appeared in the literature (*1, 12, 13, 20, 8*); only a brief outline will be presented here. Taylor's description of turbulent diffusion is valid only for a homogeneous isotropic field. Since pipe flows are neither homogeneous nor isotropic, the justification for its use is that it provides a convenient framework for presenting the results of this research.

Taylor considered the diffusion of a particle with the same properties as the fluid. He described the average behavior of a large number of these particles which originate from a fixed point in the field in terms of the variation with time of the x component of the mean-squared displacement from the origin. He related this variation to the properties of the turbulence through two parameters, the intensity of the velocity fluctuations in the x direction, and the scale, which is a measure of the lifetime of the eddies responsible for turbulent diffusion. This scale is defined by

$$\tau = \int_0^\infty R\, ds \qquad (1)$$

where

$$R = \frac{\overline{u_t u_{t+s}}}{\overline{u^2}} \qquad (2)$$

All of the above averages are taken with respect to a large number of excursions of single particles through the field. The x component of the mean-squared displacement is given as a function of time by

$$\overline{X^2} = 2\,\overline{u^2} \int_0^t (t-s) R(s)\, ds \qquad (3)$$

As time approaches zero, the correlation coefficient approaches unity and the above integral approaches

$$\overline{X^2} = \overline{u^2}\, t^2 \qquad (4)$$

If the time interval is long compared with the Lagrangian time scale, Equation (3) becomes

$$\overline{X^2} = 2\,\overline{u^2}\,\tau\, t + \text{constant} \qquad (5)$$

or

$$\frac{1}{2}\left(\frac{d\overline{X^2}}{dt}\right)_{t>>\tau} = \overline{u^2}\,\tau \qquad (6)$$

The above equation is analogous to the Einstein equation (*2*) for the molecular diffusion coefficient

$$\frac{1}{2}\frac{d\overline{X^2}}{dt} = \text{constant} = D \qquad (7)$$

However unlike Equation (6) the last expression is always valid. A turbulent diffusivity may be defined analogous to the molecular diffusion coefficient as

$$E = \overline{u^2}\,\tau \qquad (8)$$

The relation between $\overline{X^2}$ and t for intermediate times depends on the form of the correlation coefficient. As yet turbulence theory is unable to predict the dependency of the correlation coefficient on time. Likewise its experimental determination is quite difficult. Three forms for R which have been used are

$$R(s) = e^{-\alpha s} \qquad (9)$$

$$R(s) = e^{-\alpha s^2} \qquad (10)$$

$$R(s) = 1 - \alpha s \quad s < \frac{1}{\alpha} \qquad (11)$$

$$R(s) = 0 \quad s > \frac{1}{\alpha}$$

First consider the form $R = e^{-\alpha s}$. From the definition of τ the constant α can be evaluated:

$$\tau = \int_0^\infty e^{-\alpha s}\, ds = \frac{1}{\alpha}$$

$$\alpha = \frac{1}{\tau} \text{ and } R(s) = e^{-\frac{s}{\tau}} \qquad (12)$$

Substituting into Equation (3) and integrating one obtains

$$\overline{X^2} = 2\,\overline{u^2}\,\tau\left[t - \tau\left(1 - e^{-\frac{s}{\tau}}\right)\right] \qquad (13)$$

For times large in comparison with τ

$$\overline{X^2} = 2\,\overline{u^2}\,\tau t - 2\,\overline{u^2}\,\tau^2 \qquad (14)$$

If $R = e^{-\alpha s^2}$ is employed, the following equations are obtained:

$$\alpha = \frac{\pi}{4\tau^2};\quad R(s) = e^{-\frac{\pi s^2}{4\tau^2}} \qquad (15)$$

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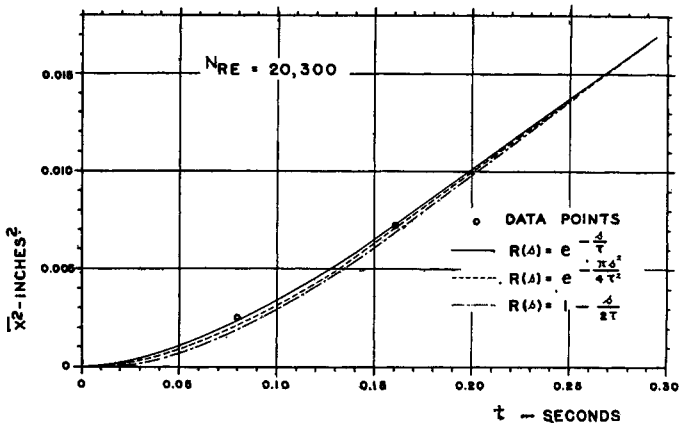

Fig. 1. Comparison of equations for the correlation coefficient at small times.

$$\overline{X^2} = 2\,\overline{u^2}\,\tau\left[\, t\,\mathrm{erf}\left(\frac{\sqrt{\pi}t}{2\tau}\right) + \frac{2\tau}{\pi}\exp\left(\frac{-\pi}{4}\frac{t^2}{\tau^2}\right) - \frac{2\tau}{\pi}\right] \quad (16)$$

$$\overline{X^2} = 2\,\overline{u^2}\,\tau t - \frac{4}{\pi}\overline{u^2}\,\tau^2 \quad t >> \tau \quad (17)$$

For the third form of the correlation coefficient considered

$$\alpha = \frac{1}{2}\tau;\ R(s) = 1 - \frac{s}{2\tau} \quad 0 \leq s \leq 2\tau$$
$$R(s) = 0 \quad s \geq 2\tau \quad (18)$$

$$\overline{X^2} = \overline{u^2}\left(t^2 - \frac{t^3}{6\tau}\right) \quad 0 \leq t \leq 2\tau \quad (19)$$

$$\overline{X^2} = 2\,\overline{u^2}\,\tau t - \frac{4}{3}\overline{u^2}\tau^2 \quad 2\tau \leq t \quad (20)$$

Equations (12) and (18) are known to be incorrect for very small values of s, since the correlation coefficient should vary as s^2 at small times (20). However despite this inaccuracy they have been used as empirical descriptions of the correlation coefficient.

Equations (13), (16), (19), and (20) are plotted in Figure 1; only two data points are shown, since most of the measurements reported in this paper were made at larger times, or larger distances from the injector, than are represented by the abscissa. The parameters τ and $\overline{u^2}$ used in calculating the curves were obtained from the slope and intercept of the asymptote to the experimental data at large times. It can be seen that the form of the $\overline{X^2}$ vs. time curve is not sensitive to the form of the correlation coefficient. However the turbulent intensity calculated by fitting Equations (13), (16), (19), and (20) to experimental data will vary somewhat depending on the correlation coefficient used. For the curves in Figure 1 $R = e^{-s/\tau}$ gives $\sqrt{\overline{u^2}}/U_c = 0.057$, $R = e^{-(\pi s^2)/(4\tau^2)}$ gives $\sqrt{\overline{u^2}}/U_c = 0.046$, and $R = 1 - s/2\tau$ gives $\sqrt{\overline{u^2}}/U_c = 0.045$. Very accurate measurements at small diffusion times would be needed to discern which of the assumed correlation coefficients best describes point-source diffusion. Such accuracy is difficult to attain in mass diffusion studies, and therefore it is not likely that such studies will establish the correct form of the correlation coefficient.

APPARATUS FOR STUDYING DIFFUSION IN AIR

Point source turbulent diffusion in air was studied by injecting hydrogen or carbon dioxide in the center of a flowing air stream. Figure 2 shows a schematic diagram of the equipment used in this study. The test section was made of two lengths of 3½-in. O.D. and 3.15-in. I.D. smooth copper pipe which were connected by a soldered collar to give a total length of 20 ft. This gave an entry section of about 72 pipe diameters prior to the test section to allow for full development of the velocity profile. The pipe was positioned in a vertical direction, and air was supplied to the top from a gear type of rotary pump. High flow rates were measured by a venturi meter attached to the lower end of the copper pipe, and low flow rates were measured with an orifice meter. Hydrogen or carbon dioxide was metered and fed to a ¼-in. steel tube extending through a fitting at the top of the 20-ft. pipe section so that the velocity of the injected gas would be approximately equal to the air velocity in the pipe. Riders consisting of three knife-edged legs were attached to the steel tubing to position it in the center of the pipe. An 8-in. length of ⅛-in. brass tubing with an inside diameter of 0.060 in. was soldered to the end of the injector tube. An additional 1½ in. of 1/16-in. hypodermic tubing was attached to the ⅛-in. brass tubing. For a few runs a total of 9½ in. of ⅛-in. brass tubing was used as the final section of the injector. The tips of the injectors were cut off at a right angle and beveled from the inside so that a sharp protruding edge was presented to the flow. The distance between the plane of the injector and the plane of the sampling tube was varied by moving the injector tube up and down the center of the pipe. A scale was etched directly on the top portion of the ¼-in. steel tube so that the exact position of the injector could be determined. Two sampling probes which could move across diameters at 90 deg. to each other were located 1 ft. from the lower end of the copper pipe. The probes were attached to a carriage that was moved by a micrometer screw. The probes consisted of a length of ⅛-in. brass tubing attached to the carriages and 1½-in. extensions silver soldered at right angles to the brass tubing. The 1½-in. extensions were either 1/16-in. hypodermic tubing or 1/8-in. thin walled nichrome tubing. The ends of the extensions were cut at right angles and were beveled in the same manner as the injector tubes.

Gas samples were withdrawn through the sampling tubes at a carefully controlled velocity which was less than the velocity of the air in the pipe. The gas composition was measured with a thermal conductivity bridge. Chambers containing thermistor elements constituted two legs of the bridge circuit. Pure air obtained from a fixed probe located halfway up the test section was passed through one of these chambers; the gas sample to be analyzed was passed through the other chamber. The gas flows were carefully controlled at a constant value and were metered by measuring the pressure drop over a constriction. These constrictions and the chambers containing the thermistor elements were immersed in a constant-temperature bath controlled to ± 0.005°C. The line leading into the chambers containing the thermistors consisted of several coils of ¼-in. copper tubing which were immersed in the constant-temperature bath. The variation of the relative resistance of the two thermistors was measured by the off balance of the bridge circuit. Since the two thermistor elements were not exactly matched, a variable resistance in series with one of the thermistors was adjusted so that the bridge was balanced when air was in both thermistor

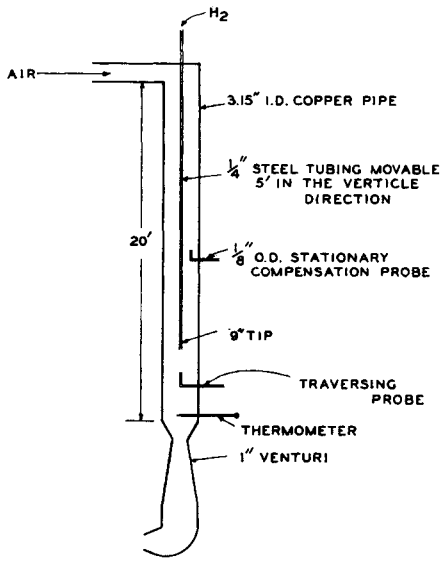

Fig. 2. Apparatus for studying diffusion in air.

chambers. The other two legs of the bridge were a 25-kohm. resistor and a 25-kohm. heliopot. The voltage source to the bridge was two 12-v. storage cells. The current was controlled at a fixed value of 1.300 ma. by a variable resistance in series with the storage cells and measured by a potentiometer from the voltage drop across a 10-ohm resistor. The off balance of the bridge was measured with a potentiometer which had a range of 0 to 80 mv. By use of the heliopot this range was extended to 8,000 mv. without changing the sensitivity, and a much larger concentration range could be conveniently measured. The heliopot is a precision variable resistor on which the variable tap can be moved by definite increments (*4*). The constriction used to meter the hydrogen and carbon dioxide was calibrated by a water-displacement method. This calibration was used as a basis for the calibration of the orifice and venturi meters and of the conductivity cells.

APPARATUS FOR STUDYING DIFFUSION IN WATER

The basic design of the apparatus for studying point source turbulent diffusion in water was the same as that used for air. A schematic drawing of the experimental setup is shown in Figure 3. The study was carried out in a 3-in. I.D. vertical Pyrex pipe. One normal potassium chloride solution was injected into the system at approximately the same velocity as the water in the pipe through a tube which could be moved up and down the center of the pipe. A length of about 75 pipe diameters preceded the test section so that the flow was fully developed in the test section. Samples were removed from the flow stream at different locations along a diameter by a stainless steel tube positioned by a micrometer screw and were analyzed for potassium chloride content by measuring their electroconductivity. Demineralized water was used to increase the sensitivity of the electroconductivity measurements. As it was impractical to discharge the water after sampling, the water was recirculated through the system. Mixed average samples of the inlet feed were taken throughout a run, and the electroconductivity of this sample was subtracted from the electroconductivity of the probe sample to give a net conductivity due to the newly injected potassium chloride solution.

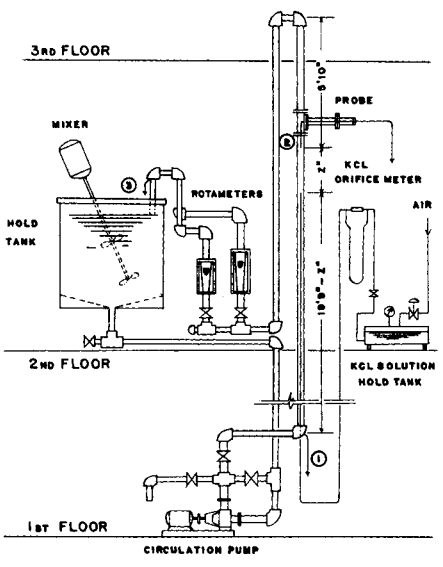

Fig. 3. Equipment for studying diffusion in water.

Table 1. Concentration Profile Data for Diffusion of Hydrogen and Carbon Dioxide

Reynolds number = 72,000
Inside pipe diameter = 3.15 in.
Gas temperatures = 28.5 to 33.7°C.
Gas pressures = 77.8 to 79.0 cm. Hg
r_c = center of the concentration profile
Z = distance between the plane of the injector and the plane of the traverse

Run I-102 Z = 5 in. r_c = 0.0172 in.		Run II-92 Z = 5 in. r_c = 0.032 in.	
r, in.	Vol. % H_2	r, in.	Vol. % CO_2
0.0000	3.69	0.0000	2.65
0.0500	3.65	0.0250	2.55
0.1000	3.08	0.0500	2.22
0.1250	2.70	0.0750	1.98
0.1500	2.31	0.1000	1.65
0.1750	1.89	0.1250	1.31
0.2000	1.487	0.1500	1.05
0.2250	1.110	0.1750	0.73
0.2500	0.805	0.1875	0.61
0.2750	0.584	0.2000	0.49
0.3000	0.400	0.2125	0.49
0.3250	0.268	−0.2750	0.49
0.3500	0.168	−0.2625	0.56
0.4000	0.060	−0.2500	0.66
0.4500	0.014	−0.2250	0.89
0.5000	0.005	−0.2000	1.20
−0.4500	0.000	−0.1750	1.53
−0.4000	0.019	−0.1500	1.85
−0.3500	0.093	−0.1250	2.17
−0.3000	0.235	−0.1000	2.40
−0.2750	0.355	−0.0750	2.61
−0.2500	0.529	−0.0500	2.71
−0.2250	0.762	−0.0250	2.71
−0.2000	1.025		
−0.1750	1.333		
−0.1500	1.704		
−0.1250	2.09		
−0.1000	2.49		
−0.0750	2.85		
−0.0500	3.25		
−0.0250	3.56		

The water was circulated with a pump and metered by two rotameters. Two rubber-tube connectors, attached to the main flow line at the outlet of the glass lined holding tank, absorbed the vibration of the holding tank caused by the mixer mounted on the tank. The injector tube was made of 5/64-in. I.D., ¼-in. O.D. stainless steel tubing. Several fins attached to the injector fixed it at the center of the pipe. A fine glass tube of 0.0327-in. I.D. and 0.070-in. O.D. was attached to the top of the stainless steel tube. The potassium chloride solution was fed to the injector from a pressurized tank lined with acid-proof paint. The sampling probe was constructed of the same size stainless steel tubing as the injector. A fine glass tube having an opening of 0.076 in. was attached to it.

The equipment used for electroconductivity measurements consisted of a bridge, a conductivity cell, an oscillator, and an oscilloscope. The four ratio arms of the bridge consisted of two fixed resistors, one adjustable standard resistor, and the conductivity cell dipped in the sample of unknown potassium chloride concentration. The resistance of the solution between two electrodes of the conductivity cell was measured by this bridge, and the conductivity could be calculated from this resistance. Alternating voltage of 1,000 cycles/sec. and about 24 v. was applied to the bridge, and the bridge was balanced by regulating the adjustable resistor. The balance point was detected by an oscilloscope which showed a flat image when the bridge was balanced. The components consisted of a shielded ratio box, an adjustable air capacitor, and a five-dial shielded a.c. resistance box. The samples were immersed in a constant-temperature bath during measurements.

TREATMENT OF DATA

Under the conditions of the experiments diffusion in the longitudinal direction had a negligible effect upon the concentration profiles (*4, 5, 6*). Likewise measurements were confined to the central portion of the pipe, where the average fluid velocity was approximately constant. Therefore the material in a plane at a distance Z from the injector could be considered as being in the field a time $t = Z/U_c$. All of the profiles except those for gas flows 2 in. from the source or closer were Gaussian. Data on a few traverses are tabulated in Table 1. For Gaussian profiles the concentration is described by the equation

$$c = \frac{Q}{2\pi U_c \overline{X^2}_m} e^{-\frac{r^2}{2\overline{X^2}_m}} \qquad (21)$$

$$Q = \pi a^2 V c_{AV} \qquad (22)$$

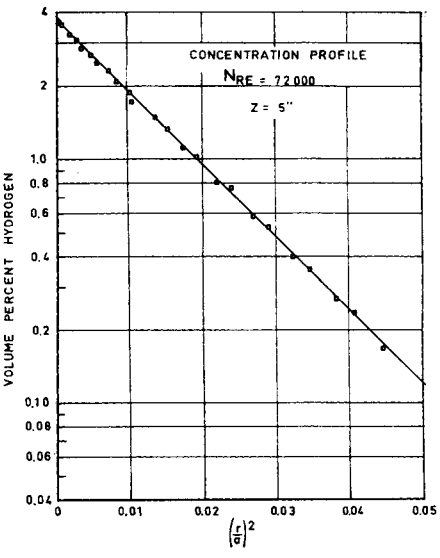

Fig. 4. Concentration profile for diffusion of hydrogen in air.

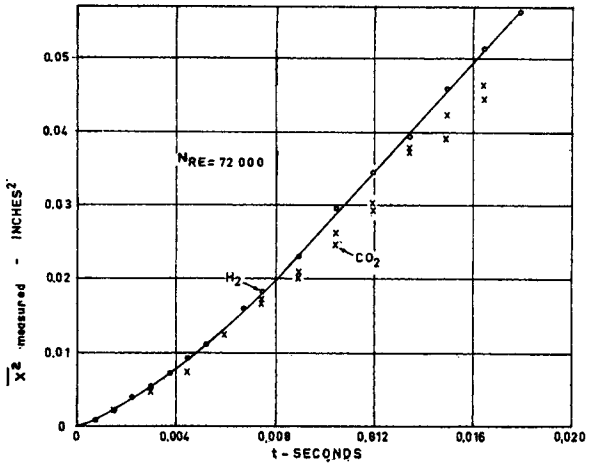

Fig. 5. Comparison of uncorrected hydrogen data with carbon dioxide data.

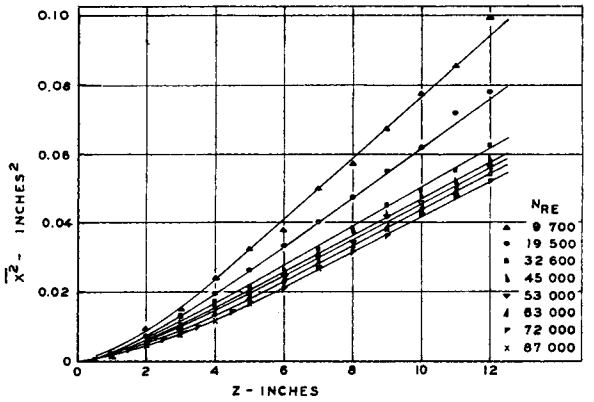

Fig. 6. Mean-square displacement data for air.

or

$$\frac{c}{c_{AV}} = \frac{a^2 V}{2 U_c \overline{X^2}_m} e^{-\frac{r^2}{2\overline{X^2}_m}} \tag{23}$$

The quantity $\overline{X^2}_m$ was calculated from the slope of a plot of either c or c/c_{AV} vs r^2. A typical plot of the data is shown in Figure 4. The mean-squared displacement could also be calculated from the intercept of such plots at $r = 0$. However the calculation from the intercept is not as reliable as the calculation from the slope, since low values of the center concentration or high values of the calculated $\overline{X^2}_m$ will result by measuring a concentration profile which does not go through the center of the diffusing plume. For the cases in which a Gaussian profile was not measured the $\overline{X^2}_m$ was evaluated by graphically integrating the following equations:

$$\overline{X^2}_m = \frac{\overline{R^2}}{2} = \frac{\int_0^a 2\pi r^3 c\, dr}{2\int_0^a 2\pi r\, c\, dr} \tag{24}$$

or

$$\overline{X^2}_m = \frac{\int_0^a r^3 \frac{c}{c_{AV}} dr}{2\int_0^a r \frac{c}{c_{AV}} dr} \tag{25}$$

From a series of concentration traverses at different distances from the injector a plot of $\overline{X^2}$ vs. t could be constructed. Taylor's description of turbulent diffusion was formulated for a point source of material. Batchelor (*1*) has shown that for finite sources the displacement can be considered as increased over the displacement for a point source by a constant amount $\overline{X^2}_o$. The quantity $\overline{X^2}_o$ is approximately equal to the displacement in the original source. If the initial source is of diameter d and if the concentration is uniform over the source

$$\overline{X^2}_o = \frac{d^2}{16} \tag{26}$$

In actual experiments it is not likely that such an ideal initial source will be obtained. In the correlation of the data for the air experiments $\overline{X^2}_o$ was calculated for a d of 1/16 in., and for the water experiments $\overline{X^2}_o$ was based on a d of 0.083 in. The values of $\overline{X^2}_o$ were small enough so that the finite injector size had a negligible effect on the plot of $\overline{X^2}$ vs. t. However the correction $\overline{X^2}_o$ did have an effect on the root mean-square displacements at small times.

The effect of molecular diffusion was negligibly small for the runs with water because of the low molecular diffusion coefficient of the diffusing substance. However in the runs with air molecular diffusion was affecting the data as can be seen in Figure 5, where data with hydrogen and carbon dioxide are compared. The carbon dioxide did not diffuse as fast as the hydrogen as can be seen by comparing the $\overline{X^2}_m$ for carbon dioxide and for hydrogen at a fixed diffusion time. An exact theory to predict the role of molecular diffusion in the turbulent-diffusion process is not yet available. Taylor (*17*) suggested that the turbulent- and molecular-diffusion processes are independent and additive:

$$\overline{X^2}_m = \overline{X^2} + 2\,Dt \tag{27}$$

This equation was used to correct all the data presented in this paper. For example by subtracting 2 Dt from the two sets of data in Figure 5 the results were brought into approximate agreement. It is expected that such a correction is valid for diffusing material with a small molecular-diffusion coefficient; however it is quite likely that it would be incorrect when the ratio of the molecular-diffusion coefficient to the turbulent-diffusion coefficient is relatively large. Hinze (*9*) discusses a theory of Burgers which suggests that turbulent diffusion is decreased by molecular diffusion owing to the escape of the diffusing property from fluid particles moving through the field. Townsend (*19*) has proposed a theory of accelerated diffusion for small diffusion times such that the spread of material would be larger than is predicted by Equation (27). Neither of these theories have been completely verified experimentally. It was not feasible to compare Townsend's theory with the data in this research, since the data at very small diffusion times were not accurate enough to justify such a comparison. Since no significant difference in the turbulent-diffusion coefficients for potassium chloride in water and for hydrogen and carbon dioxide in air were obtained with

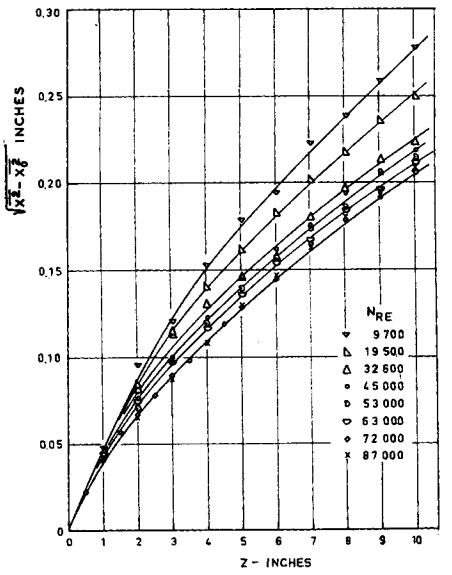

Fig. 7. Root-mean-square displacement data for air.

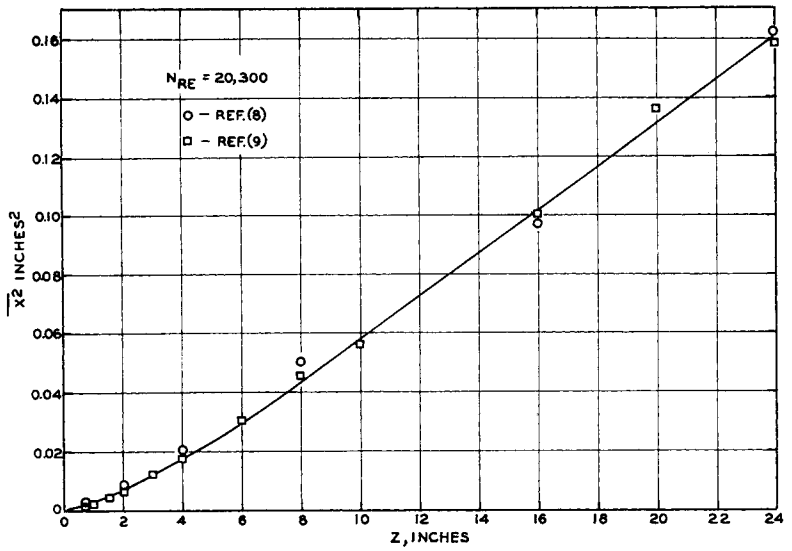

Fig. 8. Mean-square displacement data for water.

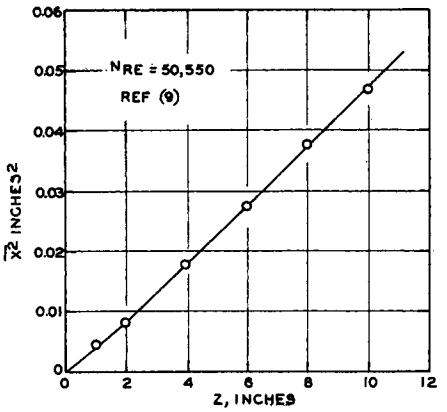

Fig. 9. Mean-square displacement data for water.

Equation (27) used to correct the data, Burger's theory could not be compared with the data.

A turbulent diffusivity, $E = \overline{u^2}\tau$, was calculated from the slope of the $\overline{X^2}$ vs. t curve at large times. From Equation (6) it can be seen that the slope is equal to twice the turbulent diffusivity.

The turbulent intensity or the Lagrangian time scale could not be evaluated from the results of this research as reliably as the turbulent diffusivity. Accurate data close to the source are needed for such an evaluation; such data are not readily obtained in mass diffusion experiments because of disturbances caused by the introduction of the diffusing substance and by the closeness of the probe and the injector. Values of $\overline{u^2}$ were calculated in two ways. From Equation (4) it can be seen that the intensity is the initial slope of a plot of $\sqrt{\overline{X^2} - \overline{X^2}_o}$ vs. t. Values of the intensity were also calculated by assuming forms for the correlation coefficient given by Equations (12), (15), and (18). From the t axis intercept of the limiting asymptote of the $\overline{X^2}$ vs. t curve and the slope of the asymptote $\overline{u^2}$ could be evaluated by means of Equations (14), (17), and (20).

RESULTS

Measurements with air with hydrogen used as the tracer gas were made for a total of eight runs which covered a range of Reynolds numbers from 9,700 to 87,000. For one Reynolds number (72,000) runs were made with both hydrogen and carbon dioxide used as the tracer gases. Measurements were made with water for Reynolds numbers of 20,300 and 50,500.

Plots of $\overline{X^2}$ vs. Z and of $\sqrt{\overline{X^2} - \overline{X^2}_o}$ vs. Z for the data with air are shown in Figures 6 and 7; plots of $\overline{X^2}$ vs. Z for water are shown in Figures 8 and 9. The two sets of data points at Re = 20,300 in Figure 8 were obtained at different times. Their agreement shows the reproducibility of the experiments.

Data points shown in Figures 6, 8, and 9 that were obtained at distances greater than 1½ to 2 pipe diameters from the injector could be approximated by the asymptotic equation for large times [Equation (5)]. As is shown in reference 4 for the air runs and in Figure 8 for the water runs the linear relation between $\overline{X^2}$ and Z extends to a distance of about 8 pipe diameters from the source. For distances greater than this the data appear to be affected by the nonuniform velocity distribution in the pipe.

Turbulent-diffusion coefficients calculated from the slopes of the $\overline{X^2}$ vs. Z curves at large Z are plotted in Figure 10 along with the data of Towle and Sherwood and of Mickelsen. Mickelsen's measurements were made in a larger pipe (8 in.) than the ones presented in this work. Owing to the dilution of the injected material he did not obtain many measurements in the region in which Equation (5) approximates the data. On the basis of the slope of the $\overline{X^2}$ vs. Z curves obtained in this research values of E were obtained from Mickelsen's results by drawing a straight line through his measurements of $\overline{X^2}$ at distances from the injector greater than 2 pipe diameters. In each of his runs only three data points could be used.

Calculated values of the turbulent intensity for air are compared in Figure 11 with the hot wire anemometer measurements of Sandborn (15) in the center of a pipe. Intensities calculated from the initial slope of the $\sqrt{\overline{X^2} - \overline{X^2}_o}$ vs. t curve and from the different forms of the correlation coefficient are higher than measurements with a hot wire. The correlation coefficient $R = e^{-t/\tau}$ gave intensities quite close to the values calculated from the initial slopes. The $\overline{X^2}$ vs. t curves were represented by $R = e^{-s/\tau}$ a little better than the other two forms tested.

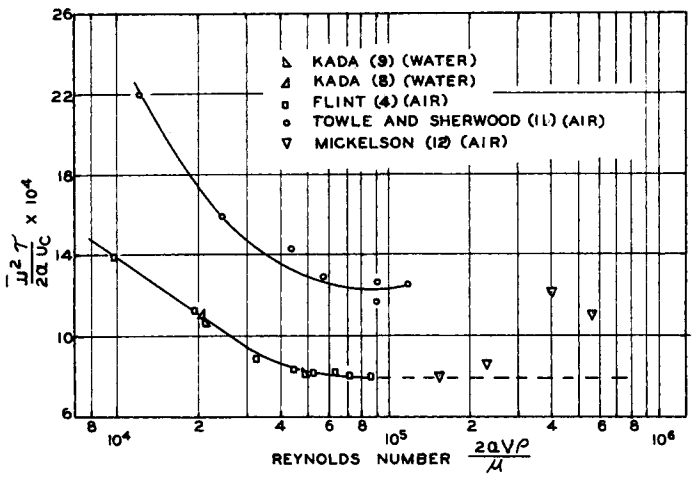

Fig. 10. Correlation of data on turbulent diffusion coefficients.

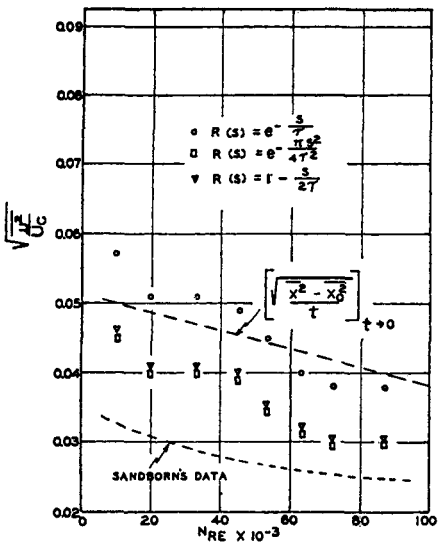

Fig. 11. Comparison of intensities with hot-wire data.

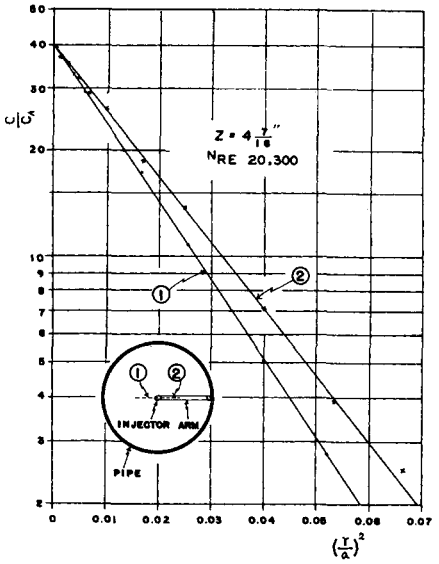

Fig. 12. Concentration profile for test injector.

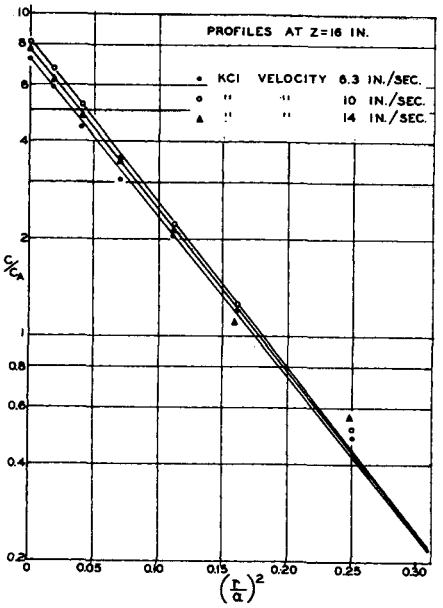

Fig. 13. Effect of injection velocity on concentration data.

DISCUSSION OF RESULTS

The good agreement between the results for air and water in Figure 10 suggests that the plot of the diffusion data in the dimensionless form of $E/2aU_c$ vs. $2aV\rho/\mu$ produces a general correlation which should be valid for other systems. The group $E/2aU_c$ decreases with Reynolds numbers and at a Reynolds number of about 50,000 levels out to a constant value of about 0.0082. The data of Mickelsen yield values of $E/2aU_c = 0.0081$ at $N_{Re} = 167{,}000$ and $E/2aU_c = 0.0087$ at $N_{Re} = 250{,}000$. His low Reynolds number runs are in fair agreement with the results of this research. However the increase in $E/2aU_c$ with increasing N_{Re} is not to be expected. Data on velocity profiles in a pipe (*16*) show that the dimensionless form of the eddy viscosity $E_m/2aU_c$ becomes approximately constant for large N_{Re}. It seems likely that the turbulent-diffusion coefficient obtained from point source diffusion measurements should show similar behavior. Therefore it is expected that the data from this research should extrapolate to higher Reynolds numbers as indicated by the dashed line in Figure 10. Since the turbulent-diffusion coefficient may be viewed as the product of an intensity term $\sqrt{\overline{u^2}}$ and a scale $\sqrt{\overline{u^2}\tau}$, the constancy of $E/2aU_c$ implies the constancy of the two terms $\sqrt{\overline{u^2}}/U_c$ and $\sqrt{\overline{u^2}\tau}/2a$. Mickelsen used an entrance section of only 36 pipe diameters prior to the test section. As he pointed out in his paper, the flow in his test section might not have been fully developed and free from entry disturbances. The data of Towle and Sherwood are about 60% higher than the results of this research. The reason for this difference is not entirely clear. A possible explanation is the lack of symmetry in the concentration profiles reported by Towle and Sherwood. They used injectors supported by horizontal arms of ¼-in. O.D. for a 6-in. duct and of ⅜-in. O.D. for a 1-ft. duct. In run series (K) (Kh) concentration profiles measured along diameters at right angles to one another were quite different. The plot of ln c vs r^2 produced a larger turbulent-diffusion coefficient for the profile in line with the injector support than for the profile perpendicular to the line of the support. However in series (B) the profiles appear to be symmetrical. Because of the possibility of a lack of symmetry in their data Towle and Sherwood used only the center-line concentrations to calculate the eddy-diffusion coefficient [see Equation (21)].

The agreement amongst the data for diffusion of hydrogen and carbon dioxide in air and for diffusion of potassium chloride in water indicates that the correction for molecular diffusion used in Equation (27) was valid for the data obtained in this research for large diffusion times and for Reynolds numbers greater than 20,000. At a Reynolds number of 20,000 the ratio of the molecular-diffusion coefficient to the turbulent-diffusion coefficient was 0.17 for the runs with hydrogen. At a Reynolds number of 9,700 this ratio would be about 0.30. Since no data were obtained with carbon dioxide or with potassium chloride at $N_{Re} = 9{,}700$, it is not certain whether Equation (27) was valid for hydrogen diffusion at this Reynolds number.

To investigate the effect of injector design some experiments were performed with the injector shown in Figure 12. This injector was a ⅛-in. O.D. brass tube. It was brought into the system through the packing in the flange just before the sampling station. The tube came down along the inside wall of the glass pipe and had two right angle bends so that the outlet faced downstream in the center of the pipe. At $N_{Re} = 20{,}300$ and at a distance of 4-7/16-in. the asymmetrical concentration profile shown in Figure 12 was obtained for a traverse along the line parallel to the support arm. Traverses perpendicular to the support arm were symmetrical and in agreement with the results obtained with the injector used in this research. It appears as if the horizontal support arm disturbed the flow.

Slight variations in the injection velocity did not appear to affect the results. This is indicated in Figure 13 by the fact that injection velocities of 6.3 in./sec., 10 in./sec., and 14 in./sec. gave approximately the same result.

The effect of the inlet condition at the bottom of the pipe used for water runs on the concentration profiles was also tested. A 1 ft. thick packed bed consisting of two layers was placed at the inlet. The lower layer was 6 in. thick and made of ¼-in. Berl saddles. The upper layer was 6-in. and consisted of ¼-in. diameter glass spheres. Diffusion data obtained with this system agreed with data obtained without a packed section at the bottom of the pipe.

In the type of experiments reported in this paper it is impossible to elimi-

nate all disturbances created in the flow stream by the injection system. It is also quite difficult to say exactly how these disturbances are affecting the results. It has been assumed that at large distances from the injector the diffusion rate is unaffected by the presence of the injector. If in these regions the concentration profile is Gaussian and is distributed symmetrically around the center of the pipe (as was true in this research), then the change in the distribution of the diffusing material will be the same as if it had originated from a point source in an undisturbed stream at some time which is different from the actual time the material has been in the field. Since the turbulent-diffusion coefficient is calculated from the slope of the $\overline{X^2}$ vs. t curve at large times, it should not be affected by disturbances created by the injector unless these disturbances are propagated a large distance downstream. Some preliminary runs (*4*) on diffusion in air with an injector twice as large as the one used to obtain the data presented in this paper yielded slightly higher (7% at $N_{Re} = 30{,}000$ and 17% at $N_{Re} = 70{,}000$) values of E than those in Figure 10. It is not certain whether this disagreement results from inaccuracies in these measurements which cannot be accounted for at this time or whether it results from increased disturbance due to the use of a larger injector. The calculation of a turbulent intensity from data at small diffusion times or from the intercept on the time axis of the asymptote to the $\overline{X^2}$ vs. t curve at large times would be affected by disturbances in the neighborhood of the injector. This is probably the reason for the lack of agreement with intensities obtained from hot wire anemometer measurements shown in Figure 10. This conclusion is supported by the results obtained by Mickelsen. He did his experiments with a pipe which was almost three times as large as the one used in this research and with an injector of the same size. Therefore the region over which Equation (4) should approximate the data was larger in his experiments, and more accurate values of the turbulent intensity should be obtained from diffusion data. He found close agreement between turbulent intensities from diffusion data and those measured with a hot-wire anemometer.

Some experiments (*2*) have been performed at a Reynolds number of 10,000 to obtain a more direct measure of the extent of the disturbances introduced by the presence of injectors of the type used in this research. A heating wire of 0.0126-in. diameter was extended across the pipe 3 in. downstream from a 20 in. length of ⅛-in. diameter tubing located in the center of the pipe. Temperature measurements made downstream of the injector were compared with measurements obtained without the tubing at the pipe center. Close to the heating wire a larger spread of heat was obtained with the tubing present. This result supports the conclusion that the discrepancy between intensities measured by the diffusion technique and by the hot-wire anemometer was due to disturbances introduced by the injector. Far downstream from the heating wire no significant difference was noted in measurements made with and without the injector being present. Figure 14 presents data at a distance of 12 in. The measurements indicate that disturbances introduced by the injector were not propagated far enough downstream to affect the calculation of the limiting slope of the $\overline{X^2}$ vs. z curve at large distances from the injector.

ACKNOWLEDGMENT

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NOTATION

a = radius of pipe
c = concentration of diffusing substance
c_{AV} = mixed average concentration of diffusing material
d = inside diameter of injector
D = molecular-diffusion coefficient
E = turbulent-diffusion coefficient $= \overline{u^2}\tau$
N_{Re} = Reynolds number $= 2aV\rho/\mu$
Q = quantity of diffusing material injected per unit time
r = radial distance
R = Lagrangian correlation coefficient

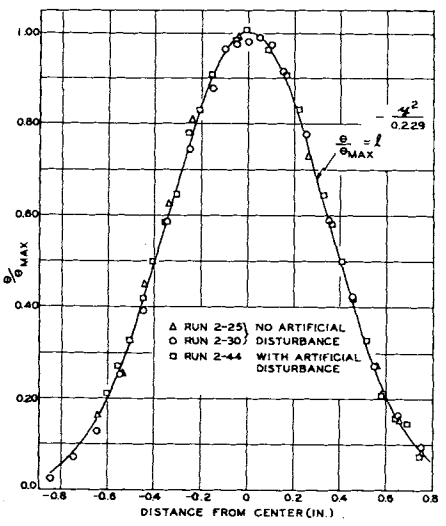

Fig. 14. Effect of artificial disturbance on diffusion of heat.

s = time increment
t = time
u = turbulent velocity fluctuation in the x direction
$\overline{u^2}$ = mean-square turbulent velocity fluctuation in the x direction
V = mixed average fluid velocity
U_c = time average fluid velocity at the center of the pipe
$\overline{X^2}$ = x component, the mean-square displacement of particles due to turbulent diffusion
$\overline{X^2_m}$ = measured value of $\overline{X^2}$
$\overline{X^2_o}$ = correction used to account for finite source
Z = distance in the direction of flow between the injector plane and the sampling plane
θ = temperature
μ = fluid viscosity
ρ = fluid density
τ = time scale characterizing turbulence $= \int_0^\infty R\,ds$

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